Modelling and Forecasting Exchange Rate movement in Nigeria in the presence of extreme influential observations

Akanbi,Olumuyiwa Olawale[1] & Alabi,Nurudeen Olawale[2]
Department of Mathematics and Statistics,
Federal Polytechnic Ilaro, Ogun State, Nigeria
Email: olumuyiwa.akanbi@federalpolyilaro.edu.ng[1] , nurudeen.alabi@federalpolyilaro.edu.ng[2]

Abstract

Influential observations or Outliers in data (both multivariate and univariate) can alter the estimates of the regression coefficients subjectively such that the underlying statistical relationships particularly in a least square estimation are rendered meaningless. In this work, we postulated an exchange rate model in the presence of extreme influential observations. Four major macroeconomic variables were included in the model as regressors. These regressors comprise foreign external reserve, foreign direct investment inflow, crude oil price and credit to private sector. Our objective is to examine the effect of extreme influential observations on the model coefficients by comparing the classical OLS estimators with the more robust least square of based on the maximum likelihood-like estimators such as MM-estimation. The analysis showed that the standard errors and p-values of the model coefficients for the robust least square are smaller than the classical OLS method. Furthermore, the forecast values from the robust MM-estimation indicate that the model slightly provide more accurate estimates than the OLS.

Keywords: influential observations, OLS, MM-estimation, exchange rate, macroeconomic variables

Introduction

Influential observations in data (both multivariate and univariate) can alter the estimates of the regression coefficients subjectively such that the underlying statistical relationships particularly in a least square estimation are rendered meaningless. In literature, several techniques have been proposed in detecting these outliers. Obafemi and Alabi (2019) opined that it is rather more difficult to detect influential observations in multivariate dataset than univariate dataset. However, in most econometric literature, detection of multivariate outliers are done via diagnostic tests when checking for model stability.

The detection and handling of these influential observations are crucial in econometric analyses because Ordinary Least Squares (OLS) estimation technique is extremely sensitive to them. Infact, the OLS estimation breakdown completely asymptotically at $1/n$ point. In order to overcome influential observations and associated problems, several techniques of estimating model parameters have been proposed in literature. These techniques are broadly classified into two; those based on diagnostic statistics (such as leverage plots and influence statistics) and the robust procedures.

Leverage plots are particularly useful graphical techniques for diagnosing any deviations of the underlying assumptions of regression models such as linearity. According to Obafemi and Alabi (2019), influential observations occur in dataset with changes in location in random directions, clusters of influential observations in particular direction or different directions. Influential observations could also occur from shift in some elements of the location vector (Rocke and Woodruff, 1996; Obafemi and Alabi, 2019). In the identification of an influential observation in a dataset, a detection specification is normally set at ± 2 standard deviations of the central tendency usually the mean (Obafemi and Alabi, 2019). Belsley, Kuh and Welsch (2004) proposed an effective technique in the identification of influential observations by plotting leverage for the k^{th} coefficient when multivariate regression are converted into a set of univariate regressions. Their work showed that the result of auxiliary regression of the residuals of original regression on the residuals of the regression of k^{th} regressor on the vector of regressors is identical to the k^{th} coefficient from the original regression. Irrespective of the dataset (univariate or multivariate), these plots are useful in detecting misspecification of the model in addition to influential observations. Alternatively, influence statistics

such as the studentized residual (RStudent), scaled difference in fitted values (DFFITS), dropped residual (DRResid), ratio of covariance matrix (COVRATIO), HatMatrix and the scaled difference in the estimated β's (DFBETAS) detect outliers or influential observations by measuring the impact of each observation on the regression output. Other methods of detecting outliers in multivariate set include those proposed by Rocke and Woodruff (1996), Barnet and Lewies (1994), Rousseeuw and Van Zomeren (1990), Maronna (1976), Campbell (1980), Stahel (1981), Donoho (1982), Rousseeuw (1985), Jackson and Chen (2004) and Obafemi and Alabi (2019).

The robust approach is the class of least squares referred to as the robust least squares (RLS) which are very efficient on multidimensional dataset. The diagnostic approaches are particularly useful when the number of influential observations are less than two observations. Inspite of the differences in the two approaches, they are less sensitive to influential observations when compared to the OLS. According to Khan *et al* (2021), the main objective of any robust estimation is to determine reliable estimates and inferences for regression model parameters in the presence of influential observations by replacing the error sum of squares in OLS with another objective function depending on the methods. These methods include Least Absolute Deviation method (LAD), *M*-estimation, *S*-estimation, and *MM*-estimation. Others are the Least Absolute Deviation (LAD) which replaces the error sum of squares with the least absolute residuals during minimization, least median square (LMS), least trimmed square (LTS) and the variance-covariance estimator of the best unit from the dataset that satisfies some optimality criteria. Like the least square estimation, LAD breaks down whenever there are high leverage points in the dataset. *M*-estimation (Huber, 1973) is more efficient than the LAD since it deals with influential observations in dependent variable in a regression model with large residuals. It achieves this by minimizing the symmetrical objective function of residuals rather than the squared errors. *S*-estimation (Rousseeuw and Yohai, 1984) on the other hand handles influential observations in the regressors in which high leverages exist. *MM*-estimation combines *S*-estimation and *M*-estimation. Its iteration starts with *S*-estimation, then uses the results to determine *M*-estimates. Ayinde, Lukman and Arowolo (2015) compared the performances of three robust least squares such as *MM*-estimation, least trimmed square (LTS) and S-estimation against the classical OLS and concluded that the estimators are mostly efficient with *M* and *MM* robust estimators.

Exchange rate movement has been a major economic growth driver in any country. Over the years, in most developed and developing countries around the world, the stability of exchange rate has been hinged on favourable economic climate. Its role in international trade cannot be overemphasized. This is particularly the case because it gives value of a country's currency interms of convertibility relative to other currencies. In Nigeria, researchers have established strong nexus between some major macroeconomic variables and exchange rate. Such macroeconomic variables include foreign external reserve, gross domestic products, industrial production, credit to private sector, crude oil price, inflation rate, interest rates, public debts, terms or balance of trade and current account deficit. Others include government intervention, speculation and political stability. Theoretically, a drop in the market price of a currency is called depreciation of the currency while a rise is known as appreciation. By implication, a rise in the exchange rate against a major currency like US Dollar, British pound, Euro etc., implies depreciation and a drop indicates appreciation of the currency. These two regimes constitute exchange rate policy and are greatly determined by government intervention through the central banks. Alabi and Bada (2022), Alabi and Ajibode (2022) postulated powerful markov switching with autoregressive dynamics models on exchange rate movement in Nigeria. Their works reveal that by modelling exchange rate using crude oil price, external reserve and inflation rate, exchange rate is more likely to transit from the appreciation regime to the depreciation regime. The analysis also show that the exchange rate is expected to spend more time in the rise than in the drop regime in Nigeria. Furthermore, strong negative relationships were established between nominal exchange rate and Nigeria's main commodity price i.e. crude oil price in both the rise and drop regimes. Similarly, foreign external reserve and inflation have significant positive influence. In a similar study by Frankel (2007), based on logit and multinomial regression models, concluded that the South African rand value is strongly dependent on the index of real prices of minerals commodities alongside interest rates, inflation rate and default risk/country risk.

Ngerebo-a and Ibe (2013) through granger causality tests revealed unidirectional causal relation from exchange rate to balance of payment, external reserve and gross domestic products in Nigeria. The study revealed that no causality existed in all directions between exchange rate, external debt and inflation rate. Kuijs (1998), through cointegration

analysis, investigated the reason for secular depreciation of the Nigeria real exchange rate experienced since the implementation of Structural Adjustment Programme (SAP) in 1986 and revealed that in a short-run dynamic setting, real exchange rate responds to shocks on balance of payments but in the long run, exchange rate is determined by the real demand and supply of foreign exchange in the economy. Suthar (2008) studied the major determinants of exchange rate in India. The research employed the classical least square (OLS) and resolved that the bank rate (RBI), short-term and log-term domestic interest differentials, interest yield differentials, and changes in the foreign external reserve all have significant impact on the average monthly exchange rate in India. Mpofu (2016) in a working paper on the volatility of exchange rate in South Africa particularly on testing the hypothesis that trade openness devalues the rand, estimated GARCH models and found that switching to floating exchange rate regime, increases the value of rand against the USDollar.

In these current work, we intend to study the effect of influential observations on model parameters by fitting two regression models (OLS and *MM*-estimation) of exchange rate movement on four macroeconomic variables such as the foreign external reserve, foreign direct investment inflow, crude oil price, and credit to private sector in Nigeria between 2006 and 2021.

This research work is divided into six sections: section one covers the introduction. In section two, we discussed briefly the effect of extreme influential observations in multivariate analysis. Section three covers the methodology which is broken into sources of data, regression analysis and robust regression through *MM*-estimation. Section four include the model specification. Section five and six comprises empirical results and discussion, and conclusion respectively.

Methods

In this section, we describe the data collected on the macroeconomic variables, the postulated models in this current study, and methods of analysis conducted to achieve the set objectives.

Sources of data

Secondary data were collected on five macroeconomic variables including nominal exchange rate, foreign external reserve, foreign direct investment inflow, crude oil price and credit to private sector. These data were sourced from the database of the Central Bank of Nigeria (CBN) between 2006 and 2021 on monthly basis, a total of 178 data points. Naira per USDollar exchange rate (nominal) was used as the response variable while, foreign direct investment inflow, foreign external reserve, crude oil price and credit to private sector were the regressors. Since the variables involved are based on different units, we converted all data to natural logarithm. This also helps to make interpretation of model coefficients easier.

Regression Analysis

Regression analysis is a powerful statistical tool that utilizes the relation between two or more quantitative variables so that one variable can be predicted from the other or others. This methodology is widely used in social and behavioral sciences such as business, biological sciences and many other disciplines (Bada and Alabi, 2022). The primary purpose of all regression analyses is to use data to estimate the form of this relation. A functional relation may be defined as

$$Y = X\beta + \varepsilon \tag{1}$$

Where Y is an $n \times 1$ column vector of the response variable, X is an $n \times p$ designed matrix of regressors, β is an $n \times (1+p)$ vector of regression coefficients and the ε is the $n \times 1$ vector of residuals. The estimators $\hat{\beta}$ are usually estimated by the minimizing the residual sum of squares otherwise called the method of least squares according to some underlying assumptions such as the normality of residuals i.e. $\varepsilon \sim N(0, \sigma^2)$.

$$\hat{\beta}_{LS} = \arg\min_{\beta} \sum_{i=1}^{N} r_i(\beta)^2 \quad (2)$$

The least square method attempt to find the value of the parameter (s) that makes the sum of squares residual to be zero. The function $r(\beta)^2 = (Y - X\hat{\beta})^2$ is very sensitive to influential variables especially on high leverage cases. At this point, the least squares breaks down and all the model's assumptions are no longer satisfied. As mention earlier, the estimation breaks down completely asymptotically at 1/*n* point.

Robust Regression through MM-estimation

The *MM*-estimation one of the most powerful statistical technique used in detecting influential observations in both the response and the regressors. This is achieved through a procedure that combines the *S*-estimation and the *M*-estimation. It begins with the *S*-estimates of the regression coefficients and $\boldsymbol{\sigma}$ used as scale. It uses the estimate of the standard error, $\boldsymbol{\sigma}$ as fixed value in iteration in order to solve the following *M*-estimation $\boldsymbol{k}$ nonlinear first-order equations:

$$\sum_{i=1}^{N} \psi_c\left(\frac{r_i(\beta)}{\sigma w_i}\right)\frac{x_{ij}}{w_i} = 0 \; j = 1, 2, \ldots\ldots\ldots, k \quad (3)$$

Where $\psi_c(\cdot) = \rho_c'(\cdot)$, the first derivative of $\rho_c(.)$ in the maximum likelihood estimator-like of the model parameters expressed as

$$\hat{\beta}_M = \arg\min_{\beta} \sum \rho_c\left(\frac{r_i(\beta)}{\sigma w_i}\right) \quad (4)$$

The residuals are scaled with $\boldsymbol{\sigma}$, $c > 0$ a tuning constant corresponding to the function ρ and w_i are down-weighted using Hat matrix and expressed as $w_i = \sqrt{1 - X_i(X'X)^{-1}X_i'}$. In the computation of $\rho_c(.)$, there are several choices such as Andrews, Bisquare, Cauchy, Fair, Huber-Bisquare, Logistic, Median, Talworth, Welsch). However, *MM*-estimation procedure requires the Bisquare given as

$$\rho_c(X) = \begin{cases} \dfrac{c^2}{6}\left(1 - \left(1 - \left(\dfrac{X}{c}\right)^2\right)^3\right) & if\,|X| \le c \\ \dfrac{c^2}{6} & elsewhere \end{cases} \quad (5)$$

The tuning constant $c = c_M$ is chosen in order achieve a desired breakdown value. *S*-estimators are coefficients of the model parameters (β), $\hat{\beta}$ with smallest estimate of the scale *S* in equation 6 which is estimated using the second stage Median Absolute Deviation, zero centered (MADZER0) approach.

$$\frac{1}{N-k}\sum_{i=1}^{N} \lambda_{c_S}\left(\frac{r_i(\beta)}{S}\right) = b \quad (6)$$

where $\lambda_{cs}(\cdot)$ with non-negative tuning constant c_S,

$$b = E_{\phi}(\lambda_{c_S}) \tag{7}$$

ϕ is standard normal with a breakdown value of $B = b/\max(\lambda_{c_S})$. The objective function in equation 6 depends on the tuning constant, c_S through b and λ_c chosen to achieve a desired B. Rousseeuw and Yohai (1984) defined a function based on the integral of the following biweight function (Bisquare function)

$$\lambda_{c_S}(X) = \begin{cases} \left(\frac{X}{c_S}\right)^6 - 3\left(\frac{X}{c_S}\right)^4 + 3\left(\frac{X}{c_S}\right)^2 & if\,|X| \le c_S \\ 1, & elsewhere \end{cases} \tag{8}$$

Salibian-Barrera and Yohai (2006) developed a Fast-*S* algorithm in the computation of the *S*-estimates. *MM*-estimation robust statistics are derived from the second *M*-estimation stage. This statistics include the coefficient of determination or measure of goodness of fit, robust Wald test to test the null hypothesis $H_0 : \hat{\beta}_M = 0$, deviance which measures the value of the objective function in equation 4 at the final coefficient estimates and estimates of scale derived from the MADZERO at the first-stage (*S*-estimation stage), robust AIC_R and BIC_B information criteria. These summary statistics are expressed in the following forms:

- $$R_W^2 = \frac{\sum_{i=1}^{N} \rho_{c_M i} \cdot (y_i - \bar{y}_W)\left(y_i - \overline{y_W}\right)}{\sqrt{\left(\sum_{i=1}^{N} \rho_{c_M i} \cdot (y_i - \bar{y}_W)\right)\left(\sum_{i=1}^{N} \left(\rho_{c_M i} \cdot \left(y_i - \overline{y_W}\right)\right)\right)}} \sim \chi^2(k_1) \tag{9}$$

  $\rho_{c_M i} = \rho_{c_M}\left(r_i(\hat{\beta})\right)/\hat{\sigma} w_i$, $\bar{y}_W = \sum_{i=1}^{N} \rho_{c_M i} \cdot y_i$ and $\overline{y_W} = \sum_{i=1}^{N} \rho_{c_M i} \cdot \hat{y}_i$

  Adjusted $R_W^2 = 1 - (1 - R_W^2)\frac{N-1}{N-k}$

- $$R_n = \hat{\beta}_1 \hat{\Sigma}_1^{-1} \hat{\beta}_1 \tag{10}$$

  $\hat{\beta}_1$ are the k_1 non-intercept robust coefficient estimates and $\hat{\Sigma}$ the estimated covariance matrix using Huber (1981) equations.

- $$Deviance = 2\hat{\sigma}^2 \sum_{i=1}^{N} \rho_{c_M}\left(\frac{r_i(\hat{\beta})}{\hat{\sigma} w_i}\right) \tag{11}$$

- $$AIC_R = 2\sum_{i=1}^{N} \rho_{c_M}\left(\frac{r_i(\hat{\beta})}{\hat{\sigma} w_i}\right) + 2k\left\{\frac{\sum_{i=1}^{N} \psi_{c_M}\left(\frac{r_i(\hat{\beta})}{\sigma w_i}\right)^2}{\sum_{i=1}^{N} \psi'_{c_M}\left(\frac{r_i(\hat{\beta})}{\sigma w_i}\right)}\right\} \tag{12}$$

- $$BIC_R = 2\sum_{i=1}^{N} \rho_{c_M}\left(\frac{r_i(\hat{\beta})}{\hat{\sigma} w_i}\right) + 2k \cdot \log(T) \tag{13}$$

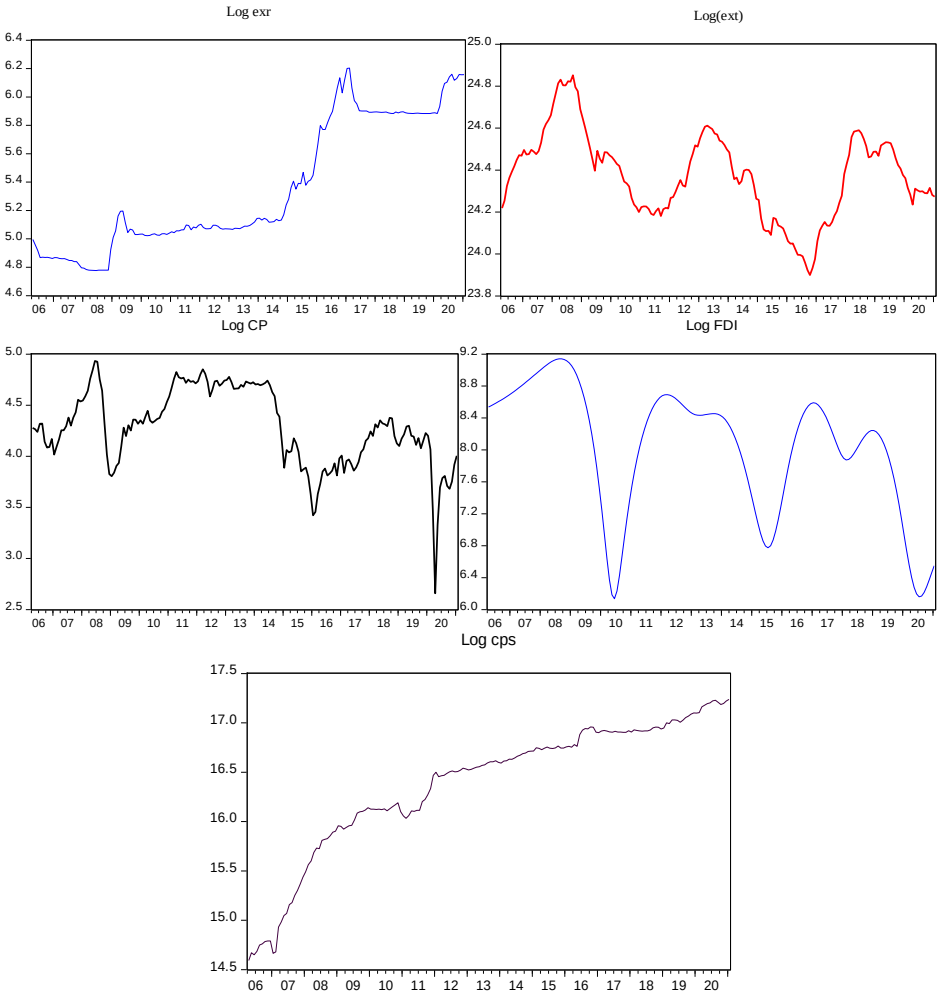
Model Specification

We specify the following multivariate regression model on exchange rate movement in Nigeria using the aforementioned five regressors (including the first lag of the response variable, *exr* to be used for the datasets as follows:

$$\log(exr_t) = \beta_1 + \beta_2 \log(exr_{t-1}) + \beta_3 \log(ext_t) + \beta_4 \log(fdi_t) + \beta_5 \log(cp_t) + \beta_6 \log(cps_t) + \varepsilon_t \quad (14)$$

Where $\log(exr_t)$ is the log of exchange rate of Naira per USDollar at time *t*, $\log(ext_t)$ is the log of the country's foreign external reserve at time *t*, $\log(\inf_t)$ is the log of the amount of foreign direct investment inflow during period *t*, $\log(cp_t)$ is the log of crude oil price time *t*, and $\log(cps_t)$ is the log of credit to private sector at period *t*. e included the first lag of the response variable as regressor in the in right hand side of the model in equation 14. Our interest is in modelling the relationship between the exchange rate and other four macroeconomic variables in which the former is the response variable and the latter the regressors (figure 1). All data are in the natural logarithm form to ensure model's linearity assumption holds.

Figure 1: Time plots on the log (exr_t), log (ext_t), log (cp_t), log (fdi_t) and log (cps_t) in Nigeria



Empirical Results and Discussion

In order to avoid the problem of spurious regression associated with econometric analyses, we check for cointegration amongst these five macroeconomic variables. If the variables are cointegrated of the same order, say, I(1), then fitting the model in equation 14 will be adequate, except that the long run comovement (long run contemporaneous relationships) amongst them will not be established. Specifically, we conducted Augmented Dickey Fuller (ADF) test on all variables. **Table** 1 shows the ADF test values and the associated *p*-values both at levels and first differences.

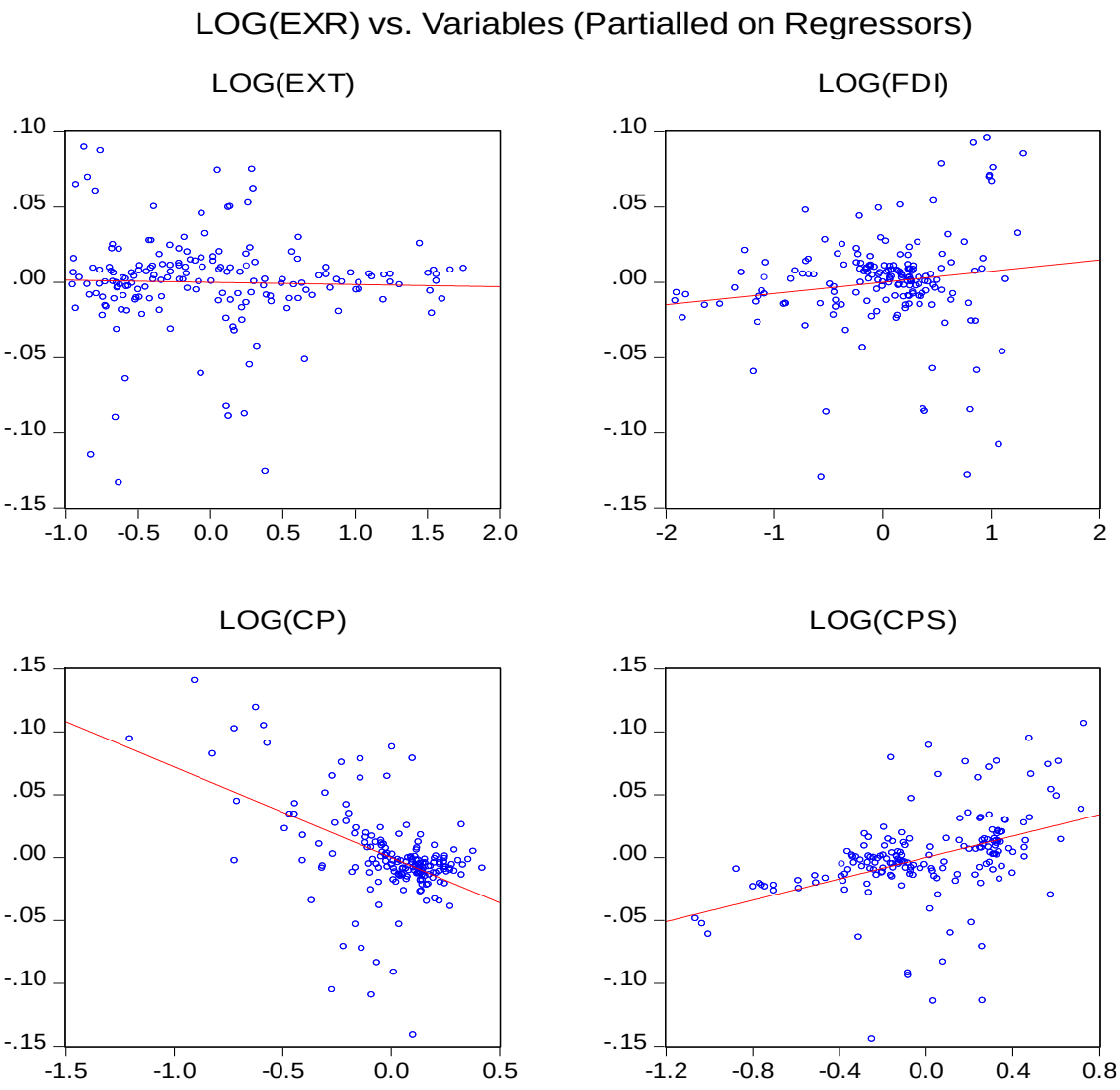
Table I: Augmented Dickey Fuller Test and *p*-values at levels

Variable	**@Level**	***p*-value**	**@1st diff.**	***p*-value**	**remark**
$\log(exr_t)$	-0.0194	0.9548	-9.5949	0.0000*	I(1)
$\log(ext_t)$	-2.2954	0.1748	-5.5755	0.0000*	I(1)
$\log(fdi_t)$	-2.3142	0.1687	-3.1266	0.0265**	I(1)
$\log(cp_t)$	-2.5071	0.1155	-9.0150	0.0000*	I(1)
$\log(cps_t)$	0.1279	0.9670	-10.9865	0.0000*	I(1)

, indicate that the significance at 1 percent and 5 percent levels of significance respectively.*

The results on ADF test show that the macroeconomic variables are nonstationary at levels but became stationary after the first difference. All *p*-values at first difference are highly significant at $\alpha = 0.01$ except the *p*-value on foreign direct investment inflow which is significant at $\alpha = 0.05$. Hence our postulated model on exchange rate in equation 14 has no problem of spurious regression. Next, we compare the ordinary least squares (OLS) with the robust *MM*-estimation method. We start with OLS estimation of the model in equation 14 by employing the HAC (Newey-West) covariance estimation procedure. This is crucial since most time series suffer from the problem of autocorrelation and heteroscedasticity, HAC estimation gives consistent estimators especially when the sample size *N* is large. The procedure corrects the standard errors of the OLS estimation in the presence of unknown autocorrelation and heteroskasticity. Stability diagnostics were conducted on this fitted OLS. Specifically, influential observations were detected using leverage plots (Figure 2).

Figure 2 Leverage Plots on Partial Regression on Exchange Rate



We observe a few observations behaving differently from others in these three plots. The plots show that the major influential observations are found between 2014 and 2017 in addition to 2008 and 2009. These points are seen to be far from the line of fit at the center of the clusters. Individually, these observations makes significant difference to the regression results obtained using the classical OLS estimation technique.

The *MM*-estimation was carried out by using the Fast-*S* algorithm. This algorithm include the *S*-estimation in which the values of the tuning constant c_s were chosen so as to achieve 99% asymptotic efficiency under residual normality. A tuning constant $c_s = 3$ and breakdown value $B = 0.2427$ were computed. The tuning value was taken from Holland and Welsch (1977). Five subsamples corresponding to the number of regressors were taken at each trail of 200. Two refinements were carried out and the best five of the 200 trials were compared. The results were then used to implement the second stage involving the *M*-estimation which has a tuning constant $c_M = 5.787$. 500 iterations were carried out using the L'Ecuyer random number generator and the seed was set to 2,138,879,884. The Huber Type III procedure was used to compute the standard errors and covariance matrix. Table II shows the results of the OLS and *MM*-estimation.

Table II: Regression estimates from classical OLS and robust *MM*-estimation

	OLS				MM_estimation			
Variable	Coeff.	S.E	*t*-value	*p*-value	Coeff.	S.E	*z*-value	*p*-value
$Log(exr_{t-1})$	0.9241	0.0132	69.8819	0.0000	0.9366	0.0094	99.4010	0.0000
$Log(ext_t)$	-0.0015	0.0034	-0.4236	0.6725	-0.0024	0.0022	-1.0953	0.2734
$Log(FDI_t)$	0.0074	0.0045	1.6551	0.0997	0.0082	0.0031	2.6255	0.0087
$Log(CP_t)$	-0.0721	0.0109	-6.6185	0.0000	-0.0701	0.0091	-7.6866	0.0000
$Log(CPS_t)$	0.0425	0.0054	7.9445	0.0000	0.0391	0.0049	8.0336	0.0000
	R^2:	0.9952	*Adj.* R^2:	0.9951	R^2:	0.8663	*Adj.* R^2:	0.8632
	AIC:	-4.029307	BIC:	-3.93959	R^2_w:	0.998064	*Adj.* R^2_w:	0.9981
	S.E Res.:	0.1742			AIC_R:	259.7826	BIC_R:	278.294

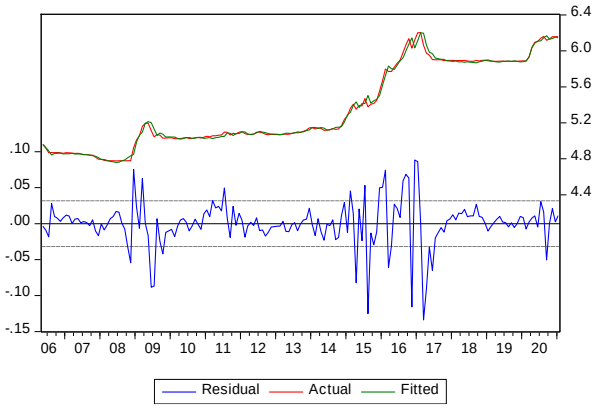
The effect of the presence of influential observations on the estimates of the model parameters can be observed in the sizes of the standard errors. The standard errors are smaller in the *MM*-estimation than in the classical OLS which produced lower *p*-values in the robust least squares estimators. The signs of the model coefficients are similar in both the OLS and robust *MM*-estimation. The results of the two models show that the foreign external reserve and crude oil price have negative impact on exchange rate movement in Nigeria. However crude oil price has a country's foreign external reserve increases by 1 percent, the exchange rate drops by 0.0015 percent implying that the Naira appreciates against the USDollar by this amount. Similarly, a percent increment in crude oil price leads to a 0.072 percent decrement in the exchange rate. This result show that every increase in the global crude oil price leads to appreciation of the Naira since sales from crude oil increases the country's forex holdings. This confirms the earlier work of Alabi and Bada (2022) in which negative relations was observed between nominal exchange rate and crude oil price in the rise and drop regimes of the Markov Switching Autoregressive model on nominal exchange rate. Also, Lawal and Alabi (2015) concluded that crude oil price has negative relationship with exchange rate in the short run in Nigeria by estimating a level relationship between exchange rate, crude oil price and inflation rate via an ARDL (4,4,0) model.

Results also show that foreign direct investment inflow and credit to private sector have significant positive effects on exchange rate in both models. By implication, increase in FDI and CPS lead to depreciation in Naira against the USDollar. In a favourable FDI environment, the relation between exchange rate and FDI inflow is always positive. The positive relationship between CPS and exchange rate implies that as credit to the private companies' increases, their demand for forex grows since most of their inputs are imported from other countries. Hence, higher demand for forex leads to depreciation of the country's currency against the USDollar. The insignificance of the country's foreign external reserve explain the possibility that external reserve has little effect on the state of the country's exchange rate. This was supported by the conclusion made by Ngerebo-a and Ibe (2013) that granger causality exists from exchange rate to external reserve and not vice versa. In the OLS and robust *MM*-estimation models, the predictors explain about 99.5 per cent and 99.81 percent of the total variability in exchange rate.

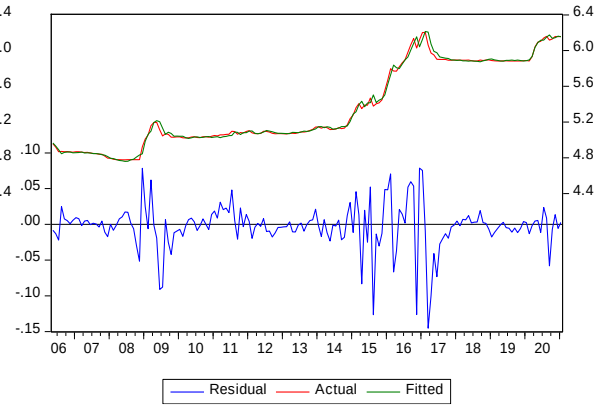
In the classical OLS model, influential observations were identified during the period under review. Like in the diagnostic approach of the influence statistics, the dates associated with these observations are 2008, 2009 and between year 2014 and 2017. When these influential observations were eliminated from the dataset, FDI becomes statistically significant but the external reserve remains insignificant. This is an indication that the presence of influential observations in datasets alter the regression estimates and results of inference on the model parameters.

Figure 4: Actual, fitted and residual plots on the two models

OLS



Robust MM-estimation



The standard descriptive statistics on the two models produced higher *R*-squared values in the classical OLS than the robust least square. However, Renaud and Victoria-Feser (2010) has shown that the robust least squares which are based on the objective function on *M*-estimation can be highly sensitive irrespective of the coefficients and the standard errors. On the other hand, the alternative R^2_w statistic provided a slightly higher value (0.9981) when compared to the OLS because the statistic is computed from a function of the residual value (equation 9). The Wald test statistic returned a value of 12,250,056 (with p-value = 0.000) and deviance of 0.1052. The forecast statistics of figure 4 indicate a Mean Absolute Error (MAE), Mean Absolute Percent Error (MAPE) are higher for OLS. Specifically, MAE_{OLS} = 5.2707, $MAPE_{OLS}$ = 1.9419. Similarly, MAE_{MM} = 5.2014, $MAPE_{MM}$ = 1.9228. These results indicate the robust least square provide slightly better estimates and hence smaller residuals.

Conclusion

Two regression models on exchange rate movement in Nigeria were estimated and compared. These models parameters were estimated using the ordinary least squares (OLS) and "maximum likelihood-like" estimators also known as the *MM*-estimation. Four macroeconomic variables such as foreign external reserve, foreign direct investment inflow, crude oil price and credit to private sector including the first lag of exchange rate were used as regressors in these two models. An initial examination of the data show a number of significant leverage points associated with influential observations. Estimation results reveal both significant and insignificant effects. In the two models, foreign external reserve was insignificant in the two models. Increases in FDI and credit to private sector lead to depreciation of the Naira against the USDollar. On the other hand, rise in crude oil price lead to significant appreciation of the country's currency. The summary statistics indicate a slightly more accurate forecast were obtained with robust *MM*-estimation.

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