DYNAMIC REGRESSION MODEL OF NIGERIA INFLATION RATE USING AN EXOGENOUS INPUT

Olubisi Aako, Gabriel Ojo & Stepen Are
Department of Mathematics & Statistics, Federal Polytechnic, Ilaro, Ogun state, Nigeria.
olubisi.aako@federalpolyilaro.edu.ng; gabriel.ojo@federalpolyilaro.edu.ng; stephen.are@federalpolyilaro.edu.ng

Abstract

This study outlines the procedural steps of an Autoregressive Integrated Moving Average (ARIMA) time series models in forecasting Nigeria inflation. A framework considering two alternative approaches such as Box Jenkins approach and the objective penalty function methods were specified by the identifying Models, evaluating the In-Sample forecasting accuracy of two statistical models deduced as ARIMAX.An inflation rate data of Sixteen (16) years were used, extracted from the Central Bank of Nigeria (CBN) website. However, the Mean Absolute Percentage Error (MAPE) and Root Mean Square Error (RMSE) serve as the error matrices in evaluating the forecast ability of the models. The effect of Akaike Information Criterion (AIC) and the linear correlation on candidate models among the considered models were tested. The ARIMAX models performed well with lower error metrics as compared to the ARIMA models in all time regimes. The linear correlation on the other hand had little or no influence at all on the in–sample forecast ability as compared to the AIC which had significant influence on the error metric. The results support that the model is efficient and hence exchange rate has effect on inflation.

Keywords: Inflation rate, Exchange rate, Framework, Evaluation, MAPE, RMSE, ARIMAX

Introduction

Generally, a stable inflation usually tends to be a prerequisite for any country sustainability as far as the economic situation in that nation is concern; which at long last takes care of its welfarism. In Nigeria therefore, inflation has remained highly unstable over the years. Since the inflation dynamism over the years, a univariate element or jointly with some explanatory variables are key in influencing its movement in the econometric stability of the country (Tamuke et al., 2018). Therefore, to forecast the inflation dynamics, some explanatory variables have to be considered in order to reduce error and improve the accuracy of a forecast.

Karadzic (2014) employed Ordinary Least Square in modeling the trade-off between import-to-export and inflation in a short run. So far, concluded that there is a direct relation between inflation and import-to-export ratio. Paul (2018) employed an autoregressive fractionally integrated moving average and a general autoregressive conditional heteroskedasticity (ARFIMA-GARCH) methodology using the ADF/KPSS to investigate the long-memory properties of CPI-Inflation. However, deduced that CPI-inflation within the study was shocked dissipating at a geometric rate in the presence of heteroskedasticity. Yang et al., (2018) in their study Compared the traditional Auto Regression Integrated Moving Average (ARIMA) model, with the Autoregressive Integrated Moving Average using Exogenous variable (ARIMAX) in forecasting the Real-time traffic. Thereafter, showed that ARIMAX model outperformed the ARIMA model. Namadina (2021) examined the use of Autoregressive Integrated Moving Average Technique in modeling and forecasting inflation rate in Nigeria. However, deduced the best output of the technique within the study. Christogonus (2021) adopted an Auto Regressive Integrated Moving Average with an exogenous variable (X-import, exchange rate, interest rate and export) in forecasting the economic growth. Their study concluded that, import performed significantly than the other counterparts of the exogenous variables for the GDP dynamics.

Many authors do not take into cognizance the effect of exogenous variables in monitoring the inflation dynamics. In this study, the main objective is to reduce forecasting error of Nigeria inflation rate. Therefore, Autoregressive Integrated Moving Average with an Exogenous input (ARIMAX) model is applied to forecast Nigeria inflation rate using interest rate as exogenous variable and compare the result with the traditional Auto Regression Integrated Moving Average (ARIMA) model.

2 Methodology

The monthly data of all items Inflation rate and Exchange rate sourced from Central Bank of Nigeria (CBN) Databank, spanned the period from January 2005 to December 2020 were used in this study and processed using RStudio.

ARIMA and ARIMAX Models

This is a more general time series lower variants of the AR or MA processes. It is the autoregressive integrated moving average process of order p, d, q, denoted as ARIMA (p, d, q). By generalizing the stationary ARMA (p, q) process for a case where one is not sure of the differencing order, one can specify,

$$(1-B)dX_t - \varphi_1(1-B)dX_{t-1} - \varphi_2(1-B)dX_{t-2} - \cdots - \varphi_p(1-B)dX_{t-p} = e_t - \theta_1 e_{t-1} - \theta_2 e_{t-2} - \cdots - \theta_q e_{t-q}$$

(1)Compactly as;

$$\Phi(B)(1-B)dX_t = \ddot{\Theta}(B)e_t \qquad (2)$$

where;

$\ddot{\Theta}(B)$ is a moving average polynomial of order q, $\Phi(B)$ is an autoregressive polynomial of order p, in the case of the ARMA process. Hence, operator $(1-B)dX_t$is the differencing operator, defined such that for d = 0, the entire process becomes the ARMA (p, q) process. However, for d = 1, equation (1) becomes the ARIMA (p, 1, q) process in equation (3),

$$(1-B)X_t - \varphi_1(1-B)X_{t-1} - \varphi_2(1-B)X_{t-2} - \cdots - \varphi_p(1-B)X_{t-p} = e_t - \theta_1 e_{t-1} - \theta_2 e_{t-2} - \cdots - \theta_q e_{t-q}$$

(3)

where the backward shift operator B operates as $B_k X_t = X_{t-k}$. While the difference between ARMA (p, q) and ARIMA (p, 1, q) processes is the first series differences (d =1) on the original time series, X_t.. Based this, ARIMAX model can take the impact of covariates into account by adding the covariate to the right hand of the ARIMA model equation, since it is an extension of Autoregressive Integrated Moving Average (ARIMA) model, presented as follows;

$$\Phi(L).\Delta^d y_t = \theta(L)\varepsilon_t \qquad (4)$$

where $\Phi(L)$is the autoregressive polynomial expressed as;

$$\Phi(L) = (1 - \Phi_1 L^1 - \Phi_2 L^2 - \cdots - \Phi_P L^P) \qquad (5)$$

and $\theta(L)$ is called the moving average polynomial expressed by;

$$\theta(L) = \left(1 - \theta_1 L^1 - \theta_2 L^2 - \cdots - \theta_q L^q\right) \qquad (6)$$

y_t, is the dependent variable in period t, Δ^ddenotes the difference from d degree and finally, $\varepsilon_t$is the white noise process. By adding the leading indicator to the equation, we have;

$$\Phi(L).\Delta^d y_t = \varphi(L)X_t + \theta(L)\varepsilon_t \quad (7)$$

where;

$$\varphi(L) = (1 - \varphi_1 L^1 - \varphi_2 L^2 - \cdots - \varphi_P L^r) \quad (8)$$

X_t is the exogenous variable at time t.

The equation;

$$\Phi(L).\Delta^d y_t = \varphi(L)X_t + \theta(L)\varepsilon_t \quad (9)$$

Equation (9) deduced the general form of ARIMAX (p,d,q) model.

Performance Model Evaluation Techniques

The selection of the best models in this study is done by using Root Mean Square Error (RMSE) for in-sample forecasting and Mean Absolute Percentage Error (MAPE) for out of sample forecasting

$$\text{RMSE} = \sqrt{\frac{\sum_{i=1}^{L}(Z_{n+l} - \hat{Z}_n(l))^2}{L}} \quad (10)$$

$$\text{MAPE} = \frac{1}{L}\sum_{L}^{L}\left|\frac{(Z_{n+l} - \hat{Z}_{n(l)})}{Z_{n+l}}\right| \times 100 \quad (11)$$

where Z_{n+l} is out-of-sample data, $\hat{Z}_{n(l)}$ is forecast value and L is the total out-of-sample data.

3 Results

Stationarity test and series differencing

Table 1: ADF Stationary Test Results following First-order Differencing

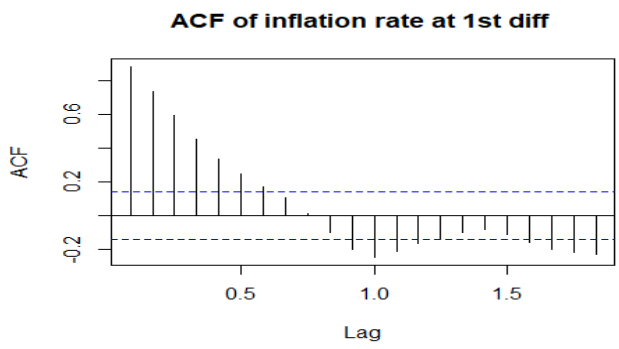
H_0: The Series has a Unit Root

Augmented Dickey-Fuller	t-Statistics	P-value
Level	**-3.4189**	**0.05315**
1st diff	**-4.326**	**0.01****

** indicates significant at 5%

Source: Extracted from R-Studio Output

Parameter Estimation



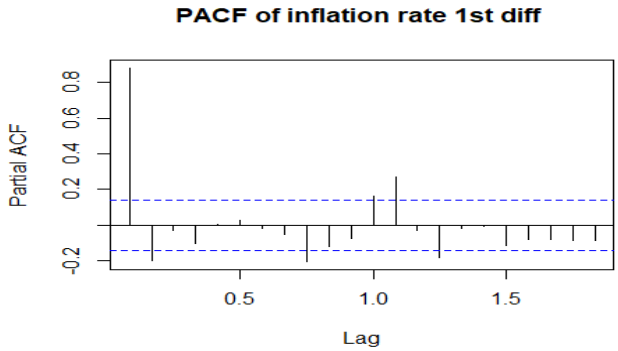


Fig.1: ACF plot of First Difference Monthly of Inflation Rate in Nigeria

Fig.2: PACF plot of First Difference Monthly of Inflation Rate in Niger

ARIMA Model Iterations

Table 2: Iterated ARIMA Models for selection using Nigeria Inflation Rate

Models	Variance	Log-likelihood	AIC
ARIMA(4,1,2)	0.03273	55.46	96.3***
ARIMA(2,1,2)	0.03408	52.63	-94.94
ARIMA(2,1,1)	0.03449	51.07	-93.92
ARIMA(1,1,2)	0.03475	50.36	-92.51
ARIMA(1,1,1)	0.03503	49.1	-92.07
ARIMA(1,1,3)	0.30487	50.54	-90.76
ARIMA(2,1,3)	0.03485	51.04	-89.62

***AIC**=Akaike Information Criterion;*
*** = Best Selected Model*
Source: Extracted from Extracted from R-Studio Output

Best fitted ARIMA (4,1,2) Model

Table 3: Iterated ARIMA Models for selection using Nigeria Inflation Rate

Parameters	Estimates	Standard Error	z value	Pr(>|z|)
AR1 (θ_1)	0.268178	0.077021	3.4819	0.0004979 ***
AR2(θ_2)	-0.081813	0.062469	-1.3097	0.1903095
AR3(θ_3)	0.734806	0.054022	13.6021	2.2e-16 ***
AR4(θ_4)	-0.269879	0.074861	-3.6051	0.0003121 ***
MA1($\emptyset_1$)	0.974547	0.042716	22.8148	2.2e-16 ***
MA2($\emptyset_2$)	0.999974	0.079088	12.6438	2.2e-16 ***

*** *indicates the significance of the coefficients @10% level*
Source: Extracted from Extracted from R-Studio Output

The model specification for ARIMA(4,1,2)in table 3 is written in form of backshift operator as;

$$(1-\phi_1 B)X_t = (1-\theta_1 B)\varepsilon_t \quad (12)$$

Substituting the coefficients, we have;

$$(1+0.268178\mathrm{B}-0.081813B+0.734806B-0.269879)X_t = (1-0.974547\mathrm{B}+0.999974\mathrm{B})\varepsilon_t \quad (13)$$

ARIMA_ARIMAX Model Estimates

Table 4: Fitted ARIMAX Model Estimates

Parameters	Estimates	Standard Error	z value	Pr(>|z|)
AR1 (θ_1)	0.27320880	0.07718564	3.5396	0.0004007 ***
AR2(θ_2)	-0.07059159	0.06550340	-1.0777	0.2811773
AR3(θ_3)	0.70889164	0.05827590	12.1644	2.2e-16 ***
AR4(θ_4)	-0.25696076	0.07577050	-3.3913	0.0006956 ***
MA1($\emptyset_1$)	0.97408148	0.0223618	43.5600	2.2e-16 ***
MA2($\emptyset_2$)	0.99999875	0.02570470	38.9033	2.2e-16 ***
ex_rate	-0.00041925	0.00034535	-1.2140	0.2247591

*** *indicates the significance of the coefficients @10% level*
. *indicates the significance of the coefficients @5% level*
Source: Extracted from Extracted from R-Studio Output

The ARIMAX model fitted is;

$$(1 + 0.27320880\mathrm{B} - 0.07059159B + 0.708891646B - 0.25696076\mathrm{B})X_t = (1 - 0.97408148\mathrm{B} + 0.9999875\mathrm{B})\varepsilon_t - 0.00041925(ex_rate) \quad (14)$$

In a more generalized linear form, we have

$$X_t = -0.27320880X_{t-1} + 0.07059159\mathrm{X}_{t-2} - 0.708891646X_{t-3} + 0.25696076\mathrm{B}X_{t-4} - 0.97408148\varepsilon_{t-1} + 0.9999875\varepsilon_{t-2} - 0.00041925(ex_rate) \quad (15)$$

where X_t is the monthly inflation rate

Diagnostic Check on the fitted ARIMA and ARIMAX models

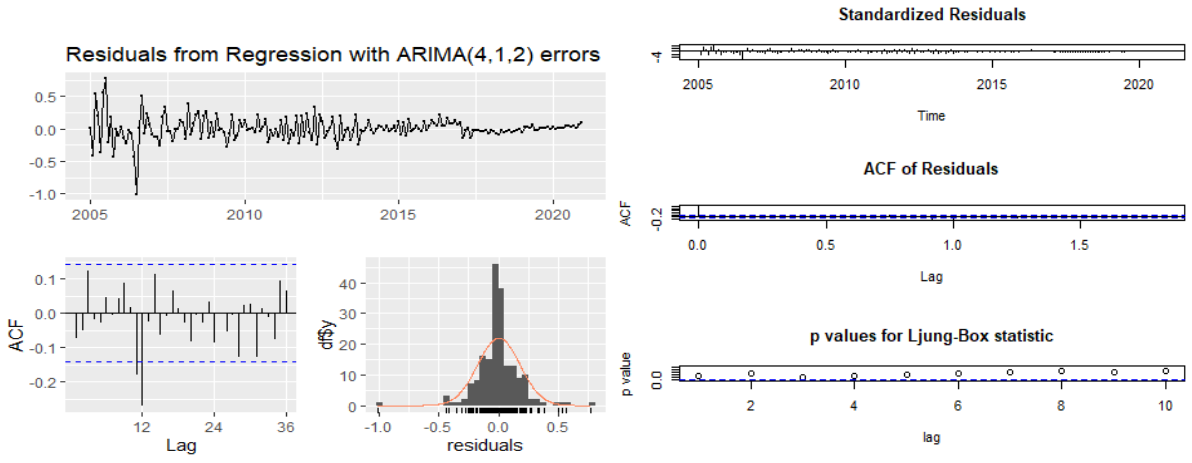


Table 4: Shapiro-Wilk Test of Residual Normality

Models	Shapiro- Wilk	p- value
ARIMA	0.89765	3.262e-10
ARIMAX	0.89808	3.464e-10

Source: Extracted from Extracted from R-Studio Output

Table 5: Ljung-Box Portmanteau Test of No serial Correlations

H_0 : No serial autocorrelations in the residuals

Models	Chi- Squared	Degree of Freedom	P- value
ARIMA	0.99719	1	0.318
ARIMAX	1.0136	1	0.314

Source: Extracted from Extracted from R-Studio Output

Forecast Comparison Result Using ARIMA and ARIMAX Model

Table 6: Forecast Evaluation for Inflation Rate

Models	MAPE	RMSE
ARIMA	1.074955	0.1775802
ARIMAX	1.070451	0.1768046

Source: Extracted from Extracted from R-Studio Output

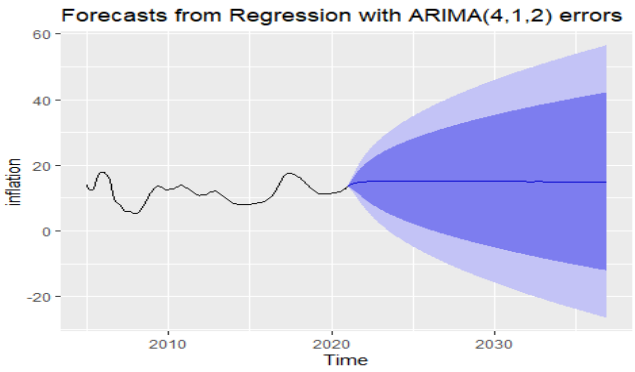


Fig 3: Forecast plot for ARIMAX model

4 Discussion

Testing the Augmented Dickey-Fuller (ADF) unit root, the stationarity of an inflation series reaffirmed the presence of unit root in inflation rate data as evidenced from the Dickey-Fuller test in Table 1 and associated P-values > 0.05 level of significance. Therefore, infer that unit root is present in the series. Calibrating the two series for model building development can only be achieved by differencing since ARIMA modeling cannot achieve white noise with a non-stationary data. First order differencing of this series achieved stationarity and the ADF test statistics was confirmed as order I(1).

Considering the Akaike information criterion (AIC), the ACF and PACF are used to ascertain the parameters q and p of the ARIMA model. However, first differenced Correlogram and Partial correlogram study in figure 1 and 2 indicates few significant spikes at the respective lags. This evidenced that the series is stationary at first difference. Hence, with the series stationarity, then model identification was deduced. Furthermore, with few iterations on this model building strategy, the orders of AR(p) and MA(q) were chosen based on the significant spikes from the ACF and PACF plots. The significant spikes in ACF of lag 1, lag 2, lag 3, lag 4, lag 5 suggest MA of order 1, 2,3 4, or 5 in figure 1. Also, the PACF with significant spikes at higher lags suggested AR of order 1, 2, 9, 12 in figure 2 respectively. These significant spikes suggested the models built for selection in table 3.3.

Table 2 indicated different ARIMA models of various orders. In order to choose the best model, we look for the model with the least AIC, σ^2 and higher log-likelihood. Analysis indicates that ARIMA order (4,1,2) is best fitted for the Inflation rate in Nigeria. Applying the rule of parsimony which says the simplest model should be chosen provided all necessary conditions are met, this resulted in choosing the ARIMA orders whose parameters were tested for significance in table 3.

Table 3 indicates that estimated parameters of AR (4), MA (2) are statistically significant at 10% level and 5% level of significance. This implies that inflation rate can well be predicted using ARIMA (4,1,2) model where order of Autoregressive and Moving Average were both selected. The inclusion of exchange rate as an exogenous variable was due to the fact of knowing if it has effect on inflation as a result of this, the inflation rate data was fitted into the ARIMA (4,1,2) as an exogenous variable to confirm its level of significance and to also know if the prediction accuracy would be higher than the generalized Box Jenkins model. Although, the order of AR and MA were found to significantly contribute to the ARIMAX model (p-value < 0.5), but the exogenous variable was found to insignificantly contribute to the model but might be adjudged the best fitted compared to the traditional ARIMA model.

The Shapiro-Wilk test of normality in table 4, has a test statistic of 0.89765 and 0.89808 for ARIMA and ARIMAX models respectively, with corresponding P-values of $3.262e^{-10}$ and $3.464e^{-10}$< 0.05 level of significance where normality of residuals of the best fitted ARIMA (4,1,2) and ARIMAX (4,1,2) with predictor variable of inflation rate was rejected at 5% significance levels. This indicates that the residuals are Normally, Independently and Identically Distributed *(N.I.I.D)*.

The large Chi-square statistic and large p-values in the Box test of the fitted ARIMA and ARIMAX residuals suggested the residuals of the models are independent. The failure in rejecting the null hypothesis that all of the autocorrelation functions are zero. In other words, we can conclude that there is no (or almost nil) evidence for non-zero autocorrelations in the residuals of the fitted model. This also indicates that the model has captured the dependence in the series. The best Inflation forecast model between ARIMA and ARIMAX model was selected based on its forecasting performance using two criteria, namely; Root Mean Squared Error (RMSE) and Mean Absolute Percent Error (MAPE). Table 6 shows ARIMAX model has the lowest values of RMSE (0.1768046) and MAPE (1.070451) as compared to ARIMA model with RMSE value of 0.1775802, and MAPE of value 1.074955. This showed ARIMAX model has performed better compared to ARIMA model and therefore, ARIMAX model was selected as the best inflation forecasting model.

5 Conclusion

This research work is an attempt to select the best and accurate model among various ARIMA and estimating the effect of exchange rate estimated models which posses' high power of predictability (forecasting power). We have identified a framework for ARIMA modeling which includes the following steps: data collection and examination; determining the order of integration; model identification; diagnostic checking; model stability testing; and forecast performance evaluation. We have adopted the traditional Box-Jenkins approach of forecasting known as ARIMA

modeling, in which a time series is expressed in terms of past values of itself (the autoregressive component) plus current and lagged values of a 'white noise' error term (the moving average component). The primary purpose behind this study was to find out which of the models (ARIMA and ARIMAX model) is more accurate and appropriate for forecasting purposes in the real-world situation, keeping in view the cost of model building. A general rule of thumb for univariate forecasting is to test at all stages of the ARIMA process. ARIMA models are theoretically justified and can be surprisingly robust with respect to alternative (multivariate) modeling approaches. The study is based on Nigeria monthly inflation data, which has used to estimate various possible ARIMA models. One of the factors that contribute to the economic growth of a country is the stability of the prices of goods and services. Ensuring price stability in the country should be the major focus and objective of the monetary policymakers. This study aimed to provide a foresight of the likely future inflation rate to the policymakers in Nigeria by modelling and forecasting the inflation rate in Nigeria. The study examined ARIMA and ARIMAX Model. Based on the AIC values, Volatility, and forecast accuracy measures, ARIMA (4,1,2) model appears as the best and most appropriate model for forecasting the monthly inflation rate in Nigeria. There by the ARIMAX (4,1,2) forecast best than the ARIMA model. The ARIMAX model was used to make a forecast of Nigeria's monthly inflation rates from January 2021 to December 2023. Among these estimated models, the best model for inflation forecast for the period 2005:01 to 2020:12 have been obtained, and using the selected ARIMA model to model an ARIMAX hereby using Exchange rate as an exogenous variable. Hence ARIMAX model forecast best the inflation rate than the ARIMA model.

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